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NONLINEAR EFFECTS IN A MAGNETIZABLE FLUID WITH INNER MOMENT OF MOMENTUM

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Nonlinear mechanical magnetic, and thermal effects are investigated in magnetizable media with inner moment of momentum in a rotating magnetic field. The moments of forces produced by the magnetizable fluid and acting on a stationary spherical vessel, the amplitude and phase shift of the electromotive force (emf) induced in a coil positioned in the magnetizable fluid, and the quantity of energy dissipated per unit volume of fluid per time unit are determined. It is shown that in a strong magnetic field hysteresis effects are always displayed by these quantities.

The hysteresis of, for instance, the moment of forces induced by the magnetizable fluid acting on the vessel is evidenced by the fact that with a slow (quasistatic) increase of the field angular rotation velocity up to some critical value, the dimensionless moment of forces monotonically decreases, and when the critical velocity is reached, it abruptly drops. As the field rotation velocity is further increased, the dimensionless moment of forces again decreases monotonically. Such abrupt spontaneous jump is also observed when the field rotation velocity is slowly decreased from some fairly high value. However the vertical value of the field angular velocity is then different. It was shown that the magnitude of the emf phase shift and the quantity of dissipated energy due to hysteresis depend on the fluid permeability.

Mathematical models of magnetizable media were proposed in [1 — 4]. These models were used in a number of investigations of magnetizable fluid behavior in a rotating magnetic field on the assumption of low velocity of field rotation, when the equations defining the medium are linear [5 — 7] and, also, without such assumption [8 — 10]. The latter had disclosed the hysteresis of a number of physical properties of the magnetizable fluid, such as magnetization intensity, the lag angle of the magnetization intensity vector relative to the magnetic field vector, the inner moment of momentum, and the macroscopic angular velocity, This effect was explained in [8] by the separation of the rotary motion of ferromagnetic suspension particles, when the vector of the particle magnetic moment lags behind the magnetic field vector by 90°.

1. Let us consider the force moment $\mathbf{m}$ induced by the magnetizable fluid and acting on a stationary spherical vessel in a uniform external magnetic field $\mathbf{H}_0$ rotating at constant angular velocity Ω_f in some plane. Absence of an inner moment of momentum in the fluid is assumed (suspension particles do not interact between themselves). On this assumption the fluid is at rest [7]. The moment $\mathbf{m}$ is determined by the formula

$$m^i = \int_\Sigma \varepsilon^{ijs} r_j (p_{sm} - \tau_{sm}) n^m d\sigma \tag{1.1}$$

where Σ is the surface of the sphere of radius R; r_j, n^m, ε^{ijs} are, respectively, the radius vector of a point of the surface, the unit vector of the inner normal to the sphere surface, and the Levi-Civita pseudotensor. The tensor τ_{ij} of Maxwellian stresses of the magnetic field outside the sphere surface and the stress tensor p_{ij} in the stationary fluid are defined in magnetostatics approximation by the formulas [4]

$$\tau_{ij} = \frac{H_i B_j}{4\pi} - \frac{H^k B_k}{8\pi}\delta_{ij}, \quad \mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \tag{1.2}$$

$$p_{ij} = -\left(p + \frac{H^k B_k}{8\pi}\right)\delta_{ij} + \frac{H_i B_j}{4\pi} + \frac{\varepsilon_{ijs} K^s}{2\tau_s}$$

where $\mathbf{K}$, $\mathbf{M}$, $\mathbf{H}$, p are, respectively, the inner moment of momentum of a unit volume, magnetization intensity, and pressure; τ_s is a constant phenomenological coefficient. The magnetostatics approximation implies that condition $R^2 \Omega_f^2 / c^2 \ll 1$, where c is the speed of light in vacuum, is satisfied.

The equations derived in [4] for the moment of momentum of a volume unit in the case, when the phenomenological coefficients are independent of the field, without taking into account cross effects and in the absence of an internal moment of momentum flux, are in the magnetostatics approximation of the form

$$\frac{\partial \mathbf{K}}{\partial t} = -\frac{\mathbf{K}}{\tau_s} + [\mathbf{M}, \mathbf{H}], \quad \frac{\partial \mathbf{M}}{\partial t} = \frac{1}{I}[\mathbf{K}, \mathbf{M}] - \frac{\mathbf{M} - \varkappa\mathbf{H}}{\tau} \tag{1.3}$$

$$\operatorname{div}(\mathbf{H} + 4\pi\mathbf{M}) = 0, \quad \operatorname{rot}\mathbf{H} = 0$$

where τ, I, $\varkappa$ are phenomenological coefficients which we assume constant. The equation of energy is separated from system (1.3), and will be considered later.

We seek a steady solution of system (1.3) on the assumption that $\mathbf{M}$ and $\mathbf{K}$ are homogeneous quantities, i.e. independent of coordinates. Vector $\mathbf{M}$ obviously lies in the plane of field rotation, and vector $\mathbf{K}$ is normal to that plane.

The assumption of homogeneous magnetization implies that the magnetizable fluid is also homogeneous and linked to the external magnetic field $\mathbf{H}_0$ by the relation [11]

$$\mathbf{H} + \frac{4\pi}{3}\mathbf{M} = \mathbf{H}_0 \tag{1.4}$$

Taking into account the homogeneity of the internal moment, from formulas (1.1) and (1.2) we obtain that the vector of moment $\mathbf{m}$ is directed along vector $\mathbf{K}$ and that their dimensionless absolute values are the same, i.e.

$$m^* = \frac{3m\tau_s}{4\pi R^3 I \Omega_f} = \frac{K}{I\Omega_f} = K^* \tag{1.5}$$

The steady solution of system (1.3) is of the form [5 — 10]

$$M_\parallel = \frac{\varkappa H}{1 + \beta^2(1 - K^*)^2}, \quad M_\perp = \beta(1 - K^*)M_\parallel, \quad \beta = \Omega_f\tau \tag{1.6}$$

where $M_{\parallel}$, $M_{\perp}$ are projections of vector $\mathbf{M}$ on directions parallel and normal to field $\mathbf{H}$, respectively. The quantity K^* is obtained from the algebraic equation

$$K^* - \frac{F(1-K^*)}{1+\beta^2(1-K^*)^2} = 0, \quad F = \frac{\varkappa H^2 \tau\tau_s}{I} \tag{1.7}$$

and the magnetic field H is obtained in the form of an implicit function from Eqs. (1.6) and (1.7) and from the relation

$$H^2\left[\left(1+\frac{4\pi M_{\parallel}}{3H}\right)^2 + \frac{16\pi^2 M_{\perp}^2}{9H^2}\right] = H_0^2 \tag{1.8}$$

which follows from (1.4).

Eliminating H from Eq. (1.7) we obtain the algebraic equation for K^*

$$K^* + \frac{\varepsilon(3+4\pi\varkappa)^2(K^*-1)}{(3+4\pi\varkappa)^2+9\beta^2(1-K^*)^2} = 0, \quad \varepsilon = \frac{\varkappa H_0^2\tau\tau_s}{I(3+4\pi\varkappa)^2} \tag{1.9}$$

The dependence of K^*, or of the equal to it quantity m^* on β^2 is determined by parameters ε and $\varkappa$, but the qualitative behavior of function $K^*(\beta^2)$

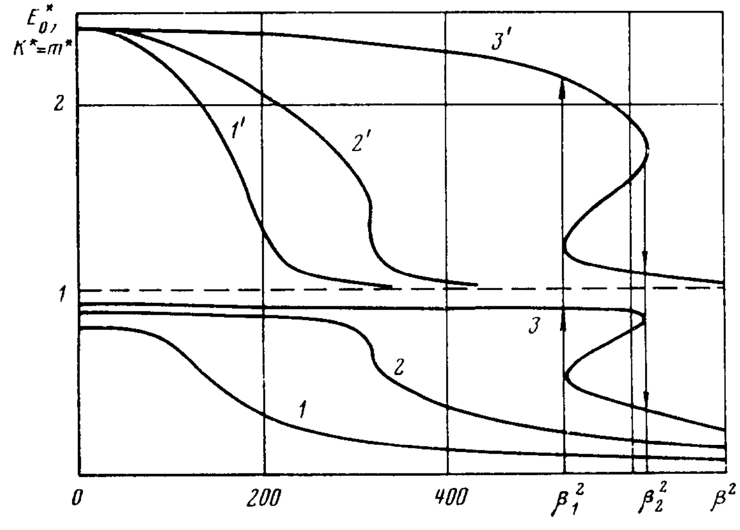

Fig. 1

depends only on ε. This dependence is shown in Fig. 1 for $\varkappa = 0.6$ and $\varepsilon < 8$ (curve *1*, $\varepsilon = 4$), $\varepsilon = 8$ (curve *2*), $\varepsilon < 8$ (curve *3*, $\varepsilon = 12$). When $\varepsilon > 8$ the internal moment K^* has in the interval (β_1^2, β_2^2) three values: two stable ones and the intermediate unstable. The cirtical values of β_1, β_2 and the respective critical values K_1^* and K_2^* of the dimensionless moment are determined from the equation $d\beta^2 / dK^* = 0$ whose solutions are

$$K_{1,2}^* = \frac{3\varepsilon \mp \sqrt{\varepsilon(\varepsilon-8)}}{4(\varepsilon+1)} \tag{1.10}$$

It follows from the foregoing that when $\varepsilon > 8$ hysteresis of the internal moment is present, and the critical values β_1 and β_2 are points of transition from one branch of curve *3* to another. The transition direction is shown in Fig. 1 by arrows. A similar result was obtained in [8] for the internal moment, but without taking into account variation of the external magnetic field in the magnetizable fluid.

2. Let us consider the effect of the emf induction in a cylindrical coil in a magnetizable fluid, with the coil axis located in the plane of field rotation. The emf E per turn of coil is defined by the Faraday induction law

$$E = -\frac{1}{c}\frac{d}{dt}\int_S (H_n + 4\pi M_n)\, d\sigma \qquad (2.1)$$

where S is the surface stretched over a single coil turn, and H_n and M_n are projections of the magnetization and magnetic field vectors on the normal to that surface.

Formulas (2.1), (1.4), (1.6), and (1.8) imply that the coil emf E varies according to a harmonic law with frequency Ω_f, while the amplitude of E_0 and the shift of phase φ are determined by formulas

$$E_0 = \frac{\Omega_f S}{c}(1 + \zeta^2)^{1/2}, \quad \zeta = \frac{4\pi M_\perp}{H + 4\pi M_\parallel} \qquad (2.2)$$

$$\varphi = \arcsin\frac{4\pi M_\perp}{3H_0} - \operatorname{arctg}\zeta$$

where S is the area of one coil turn. The phase shift φ is determined so that in the absence of magnetizable fluid $\varphi = 0$ (e.g., relative to the emf generated in the same coil oriented in the same way, but outside the vessel with fluid).

For low field rotation velocity $\Omega_f^2\tau^2 \ll 1$ formulas (2.2) assume the form

$$E_0 = \frac{\Omega_f S H_0}{3c}(3 + 4\pi\varkappa), \quad \varphi = \frac{4\pi\varkappa\beta}{\varepsilon + 1}\left(\frac{1}{3 + 4\pi\varkappa} - \frac{1}{1 + 4\pi\varkappa}\right) \qquad (2.3)$$

The parameter which qualitatively determines the dependence of the dimensionless emf $E_0^* = cE_0 / \Omega_f SH_0$ on β^2 is ε. This dependence is shown in Fig. 1 for several ε (curves $1' - 3'$, $\varkappa = 0.6$, $\varepsilon = 4, 8, 12$). These curves are analogous to curves *1—3* for the internal moment. The regression points β_1 and β_2 of curve *3* are the same as for the internal moment and are determined by solutions (1.10) in conjunction with formula (1.9). Hence the hysteresis effect is unavoidable in the coil emf when $\varepsilon > 8$.

The dependence of phase shift φ on β^2 is shown in Fig. 2 for several ε and $\varkappa$. When $\varepsilon < 8$ (curve *1*, $\varepsilon = 4$, $\varkappa = 0.1$) function $\varphi(\beta^2)$ is single-valued with its maximum at point β_0^2 determined by the condition that the respective internal moment K_0^*

$$K_0^* = \frac{\varepsilon}{\varepsilon + 2\lambda}, \quad \lambda(\varkappa) = 1 + \frac{4\pi\varkappa}{3 + 4\pi\varkappa} \qquad (2.4)$$

If $\varepsilon > 8$, the function $\varphi(\beta^2)$ in the interval (β_1^2, β_2^2) is ambiguous, and the form of the curve substantially depends on parameter $\varkappa$, and has a loop with a self-intersection point for $\lambda(\varkappa) > \lambda_c$ (curve *2*, $\varepsilon = 12$, $\varkappa = 0.07$), or is free of self-intersection when $\lambda(\varkappa) < \lambda_c$ (curve *3*, $\varepsilon = 12$, $\varkappa = 0.02$). The critical parameter λ_c is determined by formula

$$\lambda_c = (\varepsilon - \sqrt{\varepsilon(\varepsilon - 8)})/4 \qquad (2.5)$$

The foregoing implies that the quantity φ must evince the hysteresis effect when $\varepsilon > 8$, but differently for $\lambda < \lambda_c$ or $\lambda > \lambda_c$. Transition from one branch of the curve to another at slow (quasi-static) variation of the field rotation velocity is indicated in the diagram by arrows.

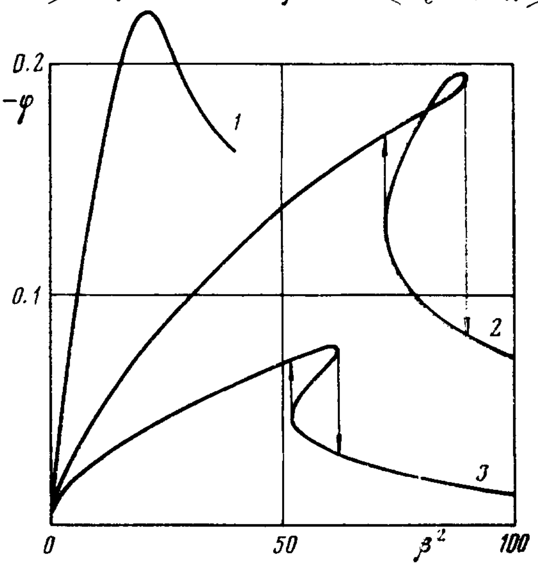

Fig. 2

3. Let us consider the energy dissipation in a magnetizable fluid in a rotating magnetic field. The energy Q dissipated per unit of volume in a unit of time is defined in the magnetostatics approximation is [4]

$$Q = \left(\mathbf{H} - \frac{\mathbf{M}}{\varkappa}\right)\left(\frac{d\mathbf{M}}{dt} - \frac{1}{I}[\mathbf{K}, \mathbf{M}]\right) + \frac{1}{I} p^{ij} \varepsilon_{ijm} K^m \qquad (3.1)$$

From formulas (3.1) and (1.2) and solutions (1.7) we have

$$Q^* = \frac{\tau Q}{H_0^2} = \frac{H^2}{H_0^2}\left[\left(1 + \frac{M_\parallel}{\varkappa H}\right)^2 + \frac{M_\perp^2}{\varkappa^2 H^2}\right] + \frac{\varkappa \beta^2 K^{*2}}{F_0}$$

which for slow field rotation velocity $\beta^2 \ll 1$ reduces to the form

$$Q^* = \frac{9\beta^2 (1 + \varepsilon\varkappa)}{(\varepsilon + 1)^2 (3 + 4\pi\varkappa)^2}$$

As $\beta^2 \to \infty$, $Q^* \to 1$ asymptotically.

The dependence of Q^* on β^2 is shown in Fig. 3 for several ε and $\varkappa$. For $\varepsilon < 8$ the dependence $Q^*(\beta^2)$ is single-valued and monotonic, if $\varepsilon\varkappa < 1$ (curve *1*, $\varepsilon = 4, \varkappa = 0.1$), or has a single maximum at point β_{00}, if $\varepsilon\varkappa > 1$ (curve *2*, $\varepsilon = 4$, $\varkappa = 0.5$). Parameter β_{00} is determined by the condition that the respective internal moment

$$K_{00}^* = (\varepsilon\varkappa - 1)/(\varepsilon\varkappa + 2\varkappa - 1)$$

If $\varepsilon > 8$ we have a section of ambiguity (β_1^2, β_2^2) in which three values of β^2 correspond to a single Q^*. The critical values of β_1 and β_2 are determined by formulas (1.9) and (1.10). The upper and lower branches of curve $Q^*(\beta^2)$ represent stable solutions, while the intermediate branch relates to an unstable solution. In this case a slow decrease or increase of the field rotation velocity is bound to produce the hysteresis effect in Q^*. Depending on $\varkappa$ its hysteresis can be of four different types that correspond to different positions of K_{00}^* in the interval (K_1^*, K_2^*).

We have for $K_{00}^* < 0$ curve 3, $\varepsilon = 12$, $\varkappa = 0.08$); for $K_1^* > K_{00}^* > 0$ curve 4, $\varepsilon = 12$, $\varkappa = 0.12$); for $K_2^* > K_{00}^* > K_1^*$ curve 5, $\varepsilon = 12$, $\varkappa = 0.6$); and for $K_{00}^* > K_2^*$ curve 6, $\varepsilon = 12$, $\varkappa = 0.71$).

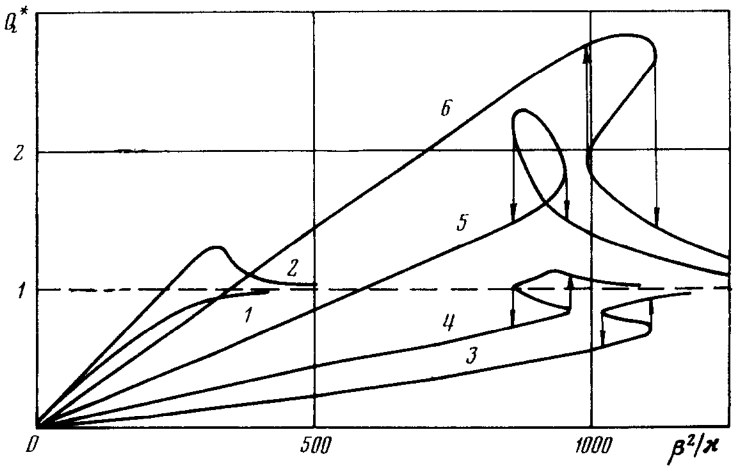

Fig. 3

Variations of the medium and field energy U per unit of volume of the medium in the absence of external heat influx to it is defined by the equation

$$dU \,/\, dt = Q \tag{3.2}$$

Let us assume that in the absence of a field and internal rotation the energy U is $U_0 = c_v T$, where T is the temperature of the medium and c_v its specific heat. The total energy U is then [4]

$$U = U_0 + \frac{M^2}{2\varkappa} + \frac{K^2}{2I} + \frac{H^2}{8\pi}$$

Taking this into account we reduce Eq. (3.2) to the form

$$dT \,/\, dt = Q/c_v$$

which shows that the temperature increase with time.

The obtained here results can be readily tested experimentally, unlike those in [8 —10]. A rotating magnetic field can be easily produced inside two orthogonal coils in which the alternating current must be of the same frequency but with phases shifted by 90°. Frequency in the region of hysteresis effects must be of the order of $10 \,/\, \tau$ (according to the estimate in [8] $\tau \sim 10^{-5}$ s). Comparison of experimental and theoretical results will make possible the determination of the phenomenological coefficients τ, τ_s, I.

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